SEM-2 (MATHEMATICS GENERAL)

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Equations of First order but not of the first degree

First we discuss degree & order of a diff. Eqn.

Degree & order of a differential Equation

The order of a differential Equation is the highest order of the differential coefficient that appears in the equation.

The degree of a differential equation is the power of the highest order differential coefficient which appears, in the equation, after the equation has been made rational and integral as far as the differential coefficients are involved.

Ex: Write the order and degree of the following differential Equations:

a) $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + x\cos y = 0$

b) $\left(\frac{d^2x}{dt^2}\right)^2 + \frac{d^2x}{dt^2} + 5\frac{dx}{dt} = 3$

c) $\frac{d^3y}{dx^3} + 5\left(\frac{dy}{dx}\right)^3 + 7y = x^2$

d) $\left(2 + \frac{d^2y}{dx^2}\right)^{3/2} = 5\frac{d^2y}{dx^2}$

Ans: a) order = 2, degree = 1
b) order = 2, degree = 2
c) order = 3, degree = 1
d) order = 2, degree = 3

[Here the equation may be written as,

$$\left(2 + \frac{d^2y}{dx^2}\right)^3 = 25\left(\frac{d^2y}{dx^2}\right)^2$$

Ex: write the order and degree of the following differential Equations:

a) $y\frac{dy}{dx} = \sqrt[3]{x}\left(\frac{dy}{dx}\right)^2 + 9$

b) $y\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + 9 = 0$ c) $5dy - 3dx = 0$

d) $x^3\frac{d^3y}{dx^3} + \cos x^2\frac{dy}{dx} + \sin x \cdot y = 0$

e) $\left(\frac{d^2y}{dx^2}\right)^{3/2} + 2\frac{dy}{dx} - y = 1$

Introduction

In this chapter we deal with differential Equations of first order and degree higher than one i.e an equation of the form $P_0p^n + P_1p^{n-1} + P_2p^{n-2} + \cdots + P_n = 0$, where $P_0(\neq 0), P_1, P_2, \cdots, P_n$ are functions of x and y, $p \equiv \frac{dy}{dx}$ and n is any +ve integer greater than one.

The equations, being of first order non-linear differential equations, may be solved after reducing the given problem to that of solving one or more equations of first order and first degree. However we consider only those equations which have one of the following types:

1) Equations solvable for $p = \frac{dy}{dx}$
2) Equations solvable for x 3) Equations solvable for y
4) Equations not containing x 5) Equations not containing y
6) Equations homogeneous in x and y
7) Lagrange's Equation 8) Clairaut's Equation

Examples of first order and not of first degree

1) $p^2 - 6p + 8 = 0$ 2) $xy(p^2+1) = (x^2+y^2)p$

3) $x = \log(p - p^2)$ 4) $y = px + p^2$